Package 'ModToppLeone'

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Type Package

Title Modified Topp-Leone Distribution: Properties, Estimation, and Applications

Version 0.1.0

Description
Provides density, cumulative distribution function, quantile function, random number generation, survival function, hazard rate function, statistical properties, classical point and interval estimation (maximum likelihood, ordinary least squares, weighted least squares, Cramer-von Mises, and maximum product of spacings), Bayesian estimation under symmetric and asymmetric loss functions (squared error, entropy, precautionary, and generalized entropy loss functions) with highest posterior density intervals, censoring schemes (random, Type-I, Type-II, and progressive Type-II censoring), and real data applications for the 'Modified Topp-Leone' distribution. Methods are based on Singh et al. (2025) <https://statassoc.or.th>, Cheng and Amin (1983) <doi:10.1111/j.2517-6161.1983.tb01267.x>, Swain et al. (1988) <doi:10.1080/00949658808811094>, Chen and Shao (1999) <doi:10.1080/1061
akrishnan and Aggarwala (2000) <doi:10.1007/978-1-4612-1178-5>.

License GPL (>= 3)

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Contents

`bayes_mtl` *Bayesian Estimation for the Modified Topp-Leone Distribution*

Description

Estimates the shape parameter `alpha` of the Modified Topp-Leone (MTL) distribution under informative (Gamma) and non-informative priors using symmetric (SELF) and asymmetric (ELF, PLF, GELF) loss functions, along with Chen-Shao Highest Posterior Density (HPD) intervals.

Usage

```
bayes_mtl(
  x,
  prior = c("informative", "noninformative"),
  a = 1,
  b = 1,
  loss = c("self", "elf", "plf", "gelf", "all"),
  delta = 1,
  n.draws = 10000,
  burn.in = 1000,
  thin = 1,
  conf.level = 0.95
)
```

Arguments

`x`	Vector of sample observations (`x > 0`).
`prior`	Type of prior distribution: `"informative"` (Gamma(a, b)) or `"noninformative"` (`a = 0`, `b = 0`).
`a`	Shape parameter for Gamma prior (hyperparameter `a > 0`).
`b`	Rate parameter for Gamma prior (hyperparameter `b > 0`).

`loss`	Loss function: `"self"` (Squared Error Loss), `"elf"` (Entropy Loss), `"plf"` (Precautionary Loss), `"gelf"` (Generalized Entropy Loss), or `"all"`.
`delta`	Constant for Generalized Entropy Loss Function (`delta != 0`). Default is 1.
`n.draws`	Number of MCMC iterations (default: 10000).
`burn.in`	Number of initial draws to discard as burn-in (default: 1000).
`thin`	Thinning interval for MCMC chain (default: 1).
`conf.level`	Credible level for HPD interval estimation (default: 0.95).

Value

A list containing point estimates under specified loss functions, posterior standard error (PSE), posterior summary statistics, MCMC sample draws, and the 100*(1 - alpha)% HPD interval.

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Chen, M. H., and Shao, Q. M. (1999). Monte Carlo estimation of Bayesian credible and HPD intervals. *Journal of Computational and Graphical Statistics*, 8(1), 69-92.

Calabria, R., and Pulcini, G. (1996). Point estimation under asymmetric loss functions for left-truncated exponential samples. *Communications in Statistics - Theory and Methods*, 25(3), 585-600.

Examples

```
set.seed(123)
sample_data <- rmtl(n = 30, alpha = 2.5)

# Bayesian Estimation under Non-informative Prior
bayes_mtl(x = sample_data, prior = "noninformative", loss = "all")

# Bayesian Estimation under Informative Prior (a = 2, b = 1)
bayes_mtl(x = sample_data, prior = "informative", a = 2, b = 1, loss = "self")
```

censoring_mtl *Censoring Schemes and Estimation for the Modified Topp-Leone Distribution*

Description

Functions to generate random samples and estimate parameters of the Modified Topp-Leone (MTL) distribution under various censoring schemes: Random Right Censoring, Type-I Censoring, Type-II Censoring, and Progressive Type-II Censoring.

Usage

```
rcensor_mtl(
  n,
  alpha,
  scheme = c("random", "type1", "type2", "progressive2"),
  T_censor = NULL,
  m = NULL,
  R = NULL,
  censor_rate = 0.5
)

mle_censor_mtl(
  x,
  status = NULL,
  scheme = c("random", "type1", "type2", "progressive2"),
  R = NULL,
  conf.level = 0.95
)
```

Arguments

n	Total initial sample size.
alpha	Shape parameter of the MTL distribution (alpha > 0).
scheme	Type of censoring scheme: "random", "type1", "type2", or "progressive2".
T_censor	Fixed censoring time for Type-I censoring (T_censor > 0).
m	Number of observed failures for Type-II or Progressive Type-II censoring (1 <= m <= n).
R	Vector of removal counts for Progressive Type-II censoring of length m such that sum(R) == n - m.
censor_rate	Rate parameter for exponential censoring distribution in random censoring.
x	Vector of observed failure/censoring times.
status	Vector of censoring indicators (1 = observed failure, 0 = censored).
conf.level	Confidence level for interval estimation (default: 0.95).

Value

`rcensor_mtl` returns a list containing observed times (`x`), censoring status (`status`), censoring scheme info, and parameter settings. `mle_censor_mtl` returns a list containing point estimate (`estimate`), standard error (`se`), asymptotic confidence interval (`ci`), and log-likelihood value under censoring.

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Balakrishnan, N., and Aggarwala, R. (2000). *Progressive Censoring: Theory, Methods, and Applications*. Birkhäuser, Boston.

Examples

```
set.seed(123)
# 1. Random Right Censoring
rand_cens <- rcensor_mtl(n = 30, alpha = 2.0, scheme = "random", censor_rate = 0.5)
mle_censor_mtl(x = rand_cens$x, status = rand_cens$status, scheme = "random")

# 2. Type-I Censoring
t1_cens <- rcensor_mtl(n = 30, alpha = 2.0, scheme = "type1", T_censor = 1.5)
mle_censor_mtl(x = t1_cens$x, status = t1_cens$status, scheme = "type1")

# 3. Type-II Censoring
t2_cens <- rcensor_mtl(n = 30, alpha = 2.0, scheme = "type2", m = 20)
mle_censor_mtl(x = t2_cens$x, status = t2_cens$status, scheme = "type2")

# 4. Progressive Type-II Censoring
R_scheme <- c(2, 0, 1, 0, 2, 0, 1, 0, 2, 2)
prog_cens <- rcensor_mtl(n = 20, alpha = 2.0, scheme = "progressive2", m = 10, R = R_scheme)
mle_censor_mtl(x = prog_cens$x, scheme = "progressive2", R = R_scheme)
```

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classical_mtl *Classical Estimation Methods for the Modified Topp-Leone Distribution*

Description

Point and interval estimation of the shape parameter `alpha` of the Modified Topp-Leone (MTL) distribution using classical methods: Maximum Likelihood (MLE), Ordinary Least Squares (OLS), Weighted Least Squares (WLS), Cramér-von Mises (CVM), and Maximum Product of Spacings (MPS).

Usage

```
mle_mtl(x, conf.level = 0.95)

ols_mtl(x)

wls_mtl(x)

cvm_mtl(x)

mps_mtl(x)

fit_mtl(
  x,
  method = c("mle", "ols", "wls", "cvm", "mps", "all"),
  conf.level = 0.95
)
```

Arguments

x	Vector of sample observations (x > 0).
conf.level	Confidence level for interval estimation (default: 0.95).
method	Estimation method: "mle", "ols", "wls", "cvm", "mps", or "all".

Value

A list or data frame containing point estimate, standard error, asymptotic confidence interval, log-likelihood, AIC, BIC, CAIC, Kolmogorov-Smirnov test statistic, and associated p-value.

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Cheng, R. C. H., and Amin, N. A. K. (1983). Estimating parameters in continuous univariate distributions with a shifted origin. *Journal of the Royal Statistical Society: Series B*, 45(3), 394-403.

Swain, J. J., Venkatraman, S., and Wilson, J. R. (1988). Least-squares estimation of distribution functions in Johnson's translation system. *Journal of Statistical Computation and Simulation*, 29(4), 271-297.

Examples

```
set.seed(123)
sample_data <- rmtl(n = 50, alpha = 2.0)
```

```
# MLE Estimation
mle_mtl(x = sample_data)

# OLS and WLS Estimation
ols_mtl(x = sample_data)
wls_mtl(x = sample_data)

# CVM and MPS Estimation
cvm_mtl(x = sample_data)
mps_mtl(x = sample_data)

# Fit all classical methods simultaneously
fit_mtl(x = sample_data, method = "all")
```

ModToppLeone *The Modified Topp-Leone Distribution*

Description

Density, distribution function, quantile function, random generation, survival function, and hazard rate function for the Modified Topp-Leone (MTL) distribution with shape parameter alpha.

Usage

```
dmtl(x, alpha, log = FALSE)

pmtl(q, alpha, lower.tail = TRUE, log.p = FALSE)

qmtl(p, alpha, lower.tail = TRUE, log.p = FALSE)

rmtl(n, alpha)

smtl(x, alpha, log.p = FALSE)

hmtl(x, alpha, log = FALSE)
```

Arguments

x, q	Vector of quantiles.
alpha	Shape parameter of the MTL distribution (alpha > 0).
log, log.p	Logical; if TRUE, probabilities/densities p are given as $\log(p)$. Default is FALSE.
lower.tail	Logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise $P[X > x]$.
p	Vector of probabilities.
n	Number of observations. If length(n) > 1, the length is taken to be the number required.

Details

The probability density function (PDF) of the Modified Topp-Leone distribution is given by:

$$f(x;\alpha) = 2\alpha(1+x)^{-(2\alpha+1)}(2x+x^2)^{\alpha-1}, \quad x > 0,\ \alpha > 0$$

The cumulative distribution function (CDF) is given by:

$$F(x;\alpha) = \left(\frac{2x+x^2}{(1+x)^2}\right)^{\alpha} = \left(1 - \frac{1}{(1+x)^2}\right)^{\alpha}, \quad x > 0,\ \alpha > 0$$

The quantile function is derived by inverting the CDF:

$$Q(p;\alpha) = (1-p^{1/\alpha})^{-1/2} - 1, \quad 0 < p < 1$$

Value

`dmtl` gives the density, `pmtl` gives the distribution function, `qmtl` gives the quantile function, `rmtl` generates random deviates, `smtl` gives the survival function, and `hmtl` gives the hazard rate function.

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Examples

```
# Evaluate PDF and CDF at x = 1 with alpha = 1.5
dmtl(x = 1, alpha = 1.5)
pmtl(q = 1, alpha = 1.5)

# Quantile function and random generation
p_seq <- c(0.25, 0.50, 0.75)
qmtl(p = p_seq, alpha = 1.5)
set.seed(123)
x_sample <- rmtl(n = 10, alpha = 1.5)
x_sample

# Survival and Hazard functions
smtl(x = 1, alpha = 1.5)
hmtl(x = 1, alpha = 1.5)
```

`plot_mtl` *Diagnostic and Visualization Plots for the Modified Topp-Leone Distribution*

Description

Plot probability density function (PDF), cumulative distribution function (CDF), survival function (SF), or hazard rate function (HRF) of the Modified Topp-Leone (MTL) distribution for specified values of `alpha`.

Usage

```
plot_mtl(
  alpha = c(0.5, 1.5, 3, 10),
  type = c("pdf", "cdf", "sf", "hrf"),
  x_max = 10,
  n_points = 500
)
```

Arguments

`alpha`	Vector of shape parameter values (`alpha > 0`).
`type`	Type of plot: `"pdf"`, `"cdf"`, `"sf"`, or `"hrf"`.
`x_max`	Maximum value of x on the horizontal axis (default: 10).
`n_points`	Number of evaluation points along the x-axis (default: 500).

Value

No return value, called for side effects (produces a plot).

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Examples

```
# Plot PDF for various values of alpha
plot_mtl(alpha = c(0.5, 1.5, 3.0, 10.0), type = "pdf", x_max = 5)

# Plot HRF for various values of alpha
plot_mtl(alpha = c(0.5, 1.5, 3.0, 10.0), type = "hrf", x_max = 5)
```

properties_mtl *Statistical Properties of the Modified Topp-Leone Distribution*

Description

Functions to calculate various statistical properties of the Modified Topp-Leone (MTL) distribution including mode, mean, quantiles, mean deviations, inequality measures, mean residual life, mean past life, probability-weighted moments, entropies, stress-strength reliability, and order statistics.

Usage

```
mode_mtl(alpha)

mean_mtl(alpha, n_terms = 100)

moments_mtl(r, alpha, n_terms = 100)

quantiles_mtl(alpha)

meandev_mtl(alpha)

inequality_mtl(y, alpha)

mrl_mtl(y, alpha)

mpl_mtl(y, alpha)

pwm_mtl(r, s, alpha)

entropy_mtl(alpha, type = c("shannon", "renyi", "omega"), gamma = 0.5)

ssr_mtl(alpha1, alpha2)

order_stats_mtl(y, alpha, n, u, type = c("pdf", "cdf"))
```

Arguments

alpha	Shape parameter of the MTL distribution (alpha > 0).
n_terms	Number of terms to use in series expansions (default: 100).
r	Order of the raw moment or probability-weighted moment.
y	Evaluation point for residual life, past life, inequality curves, or order statistics.
s	Parameter for probability-weighted moment.
type	Type of entropy ("renyi", "shannon", or "omega") or order statistics output ("pdf" or "cdf").
gamma	Order parameter for Rényi or ω-entropy (gamma > 0, gamma != 1).

alpha1, alpha2	Shape parameters for two independent MTL variables in stress-strength reliability.
n	Sample size for order statistics.
u	Index of the order statistic (1 <= u <= n).

Value

Numeric scalar or vector corresponding to the calculated statistical property:

- mode_mtl: Mode of the MTL distribution.
- mean_mtl: Mean of the MTL distribution.
- moments_mtl: r-th raw moment (finite for $r = 1$).
- quantiles_mtl: Named vector of quantiles, median, skewness, and kurtosis.
- meandev_mtl: Mean deviation about mean and median.
- inequality_mtl: Lorenz, Bonferroni, and Zenga inequality measures.
- mrl_mtl: Mean residual life function.
- mpl_mtl: Mean past life (expected inactivity time) function.
- pwm_mtl: Probability-weighted moment $\mu_{r,s}$.
- entropy_mtl: Specified entropy measure.
- ssr_mtl: Stress-strength reliability $R = P[Y_2 < Y_1]$.
- order_stats_mtl: Density or CDF of the u-th order statistic.

Author(s)

Shikhar Tyagi, Abhishek Tyagi, Arvind Pandey, Bhupendra Singh, Vrijesh Tripathi

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Examples

```
# Mode and Mean
mode_mtl(alpha = 2.5)
mean_mtl(alpha = 1.5)

# Quantiles summary
quantiles_mtl(alpha = 1.5)

# Mean deviations
meandev_mtl(alpha = 1.5)

# Inequality measures at y = 1
inequality_mtl(y = 1, alpha = 1.5)
```

```
# Mean residual life and past life
mrl_mtl(y = 1, alpha = 1.5)
mpl_mtl(y = 1, alpha = 1.5)

# Entropies
entropy_mtl(alpha = 1.5, type = "shannon")
entropy_mtl(alpha = 1.5, type = "renyi", gamma = 0.5)

# Stress-Strength Reliability
ssr_mtl(alpha1 = 2, alpha2 = 3)

# Order statistics density for median in n = 5 sample
order_stats_mtl(y = 1, alpha = 1.5, n = 5, u = 3, type = "pdf")
```

`real_datasets` *Real Datasets used in Modified Topp-Leone Research Paper*

Description

Three real-world datasets examined in Singh et al. (2025) to illustrate the flexibility and applicability of the Modified Topp-Leone distribution.

Usage

```
dataset_air

dataset_covid_india

dataset_covid_france
```

Format

Numeric vectors containing positive observations:

dataset_air Number of successive failures for the air conditioning system of each member in a fleet of 13 Boeing 720 jet airplanes (divided by 100 for fitting convenience).

dataset_covid_india Daily new COVID-19 cases in India from 16 March 2021 to 16 April 2021 (divided by 10,000).

dataset_covid_france Daily new COVID-19 cases in France from 5 August 2020 to 5 November 2020 (divided by 10,000).

An object of class `numeric` of length 39.

An object of class `numeric` of length 50.

References

Singh, B., Tyagi, S., Singh, R. P., and Tyagi, A. (2025). Modified Topp-Leone Distribution: Properties, Classical and Bayesian Estimation with Application to COVID-19 and Reliability Data. *Thailand Statistician*, 23(1), 72-96.

Cordeiro, G. M., and Lemonte, A. J. (2011). The beta-Birnbaum-Saunders distribution: An improved distribution for fatigue life modelling. *Computational Statistics & Data Analysis*, 55(3), 1445-1461.

Examples

```
data(dataset_air)
mle_mtl(dataset_air)

data(dataset_covid_india)
fit_mtl(dataset_covid_india, method = "all")

data(dataset_covid_france)
bayes_mtl(dataset_covid_france, prior = "noninformative")
```

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