

Package ‘HotellingEllipse’

September 29, 2026

Title Hotelling’s T-Squared Statistic and Ellipse

Version 1.3.0

Description Functions to calculate the Hotelling’s T-squared statistic and corresponding confidence ellipses. Provides the semi-axes of the Hotelling’s T-squared ellipses at user-defined confidence levels (95% and 99% by default). Enables users to obtain the coordinates in two or three dimensions at user-defined confidence levels, allowing for the construction of 2D or 3D ellipses with customized confidence levels. Bro and Smilde (2014) <[DOI:10.1039/c3ay41907j](https://doi.org/10.1039/c3ay41907j)>. Breton (2016) <[DOI:10.1002/cem.2763](https://doi.org/10.1002/cem.2763)>.

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LazyData true

LazyDataCompression xz

RoxygenNote 7.3.1

URL <https://github.com/ChristianGoueguel/HotellingEllipse>

BugReports <https://github.com/ChristianGoueguel/HotellingEllipse/issues>

Imports magrittr, stats, tibble

Depends R (>= 2.10)

Suggests rmarkdown, knitr, markdown, testthat (>= 3.0.0), spelling, covr, dplyr, FactoMineR, ggforce, ggplot2, glue, purrr, rgl, scales, viridisLite

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ellipseCoord	<i>Coordinate Points of the Hotelling's T-squared Ellipse</i>
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Description

This function calculates the coordinate points for drawing a Hotelling's T-squared ellipse based on multivariate data. It can generate points for both 2D and 3D ellipses.

Usage

```
ellipseCoord(  
  x,  
  pcx = 1,  
  pcy = 2,  
  pcz = NULL,  
  conf.limit = 0.95,  
  pts = 200,  
  method = c("f", "beta")  
)
```

Arguments

x	A matrix, data frame or tibble containing scores from PCA, PLS, ICA, or other dimensionality reduction methods. Each column should represent a component, and each row an observation.
pcx	An integer specifying which component to use for the x-axis (default is 1).
pcy	An integer specifying which component to use for the y-axis (default is 2).
pcz	An integer specifying which component to use for the z-axis for 3D ellipsoids. If NULL (default), a 2D ellipse is computed.
conf.limit	A numeric value between 0 and 1 specifying the confidence level for the ellipse (default is 0.95, i.e., 95% confidence).
pts	An integer specifying the number of points to generate for drawing the ellipse (default is 200). Higher values result in smoother ellipses.
method	A character string specifying how the T-squared limit is computed: "f" (default) or "beta". See ellipseParam() for details.

Details

The function computes the shape and orientation of the ellipse based on the Hotelling's T-squared distribution and the specified components. It then generates a set of points that lie on the ellipse's surface at the specified confidence level. For 2D ellipses, the function uses two components `pcx` and `pcy`. For 3D ellipsoids, it uses three components `pcx`, `pcy`, and `pcz`. The `conf.limit` parameter determines the size of the ellipse. A higher confidence level results in a larger ellipse that encompasses more data points. When the selected components are correlated, the ellipse (ellipsoid) is rotated accordingly; for uncorrelated scores, such as the PCA or PLS scores of the samples the model was fitted on, its axes are aligned with the components.

Value

A data frame containing the coordinate points of the Hotelling's T-squared ellipse:

- For 2D ellipses: columns `x` and `y`
- For 3D ellipsoids: columns `x`, `y`, and `z`

Author(s)

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Examples

```
## Not run:
# Load required libraries
library(HotellingEllipse)
library(dplyr)

data("specData", package = "HotellingEllipse")

# Perform PCA
set.seed(123)
pca_mod <- specData %>%
  select(where(is.numeric)) %>%
  FactoMineR::PCA(scale.unit = FALSE, graph = FALSE)

# Extract PCA scores
pca_scores <- pca_mod$ind$coord %>% as.data.frame()

# Example 1: Calculate Hotelling's T-squared ellipse coordinates
xy_coord <- ellipseCoord(pca_scores, pcx = 1, pcy = 2)

# Example 2: Calculate Hotelling's T-squared ellipsoid coordinates
xyz_coord <- ellipseCoord(pca_scores, pcx = 1, pcy = 2, pcz = 3)

## End(Not run)
```

ellipseParam

*Hotelling's T-squared Statistic and Ellipse Parameters***Description**

This function calculates Hotelling's T-squared statistic and, when applicable, the lengths of the semi-axes of the Hotelling's ellipse. It can work with a specified number of components or use a cumulative variance threshold.

Usage

```
ellipseParam(
  x,
  k = 2,
  pcx = 1,
  pcy = 2,
  threshold = NULL,
  rel.tol = 0.001,
  abs.tol = .Machine$double.eps,
  method = c("f", "beta"),
  conf.limit = c(0.95, 0.99)
)
```

Arguments

x	A matrix, data frame or tibble containing scores from PCA, PLS, ICA, or other similar methods. Each column should represent a component, and each row an observation.
k	An integer specifying the number of components to use (default is 2). This parameter is ignored if threshold is provided.
pcx	An integer specifying which component to use for the x-axis when k = 2 (default is 1).
pcy	An integer specifying which component to use for the y-axis when k = 2 (default is 2).
threshold	A numeric value between 0 and 1 specifying the desired cumulative explained variance threshold (default is NULL). If provided, the function determines the minimum number of components needed to explain at least this proportion of total variance. When NULL, the function uses the fixed number of components specified by k.
rel.tol	A numeric value specifying the minimum proportion of total variance a component should explain to be considered non-negligible (default is 0.001, i.e., 0.1%).
abs.tol	A numeric value specifying the minimum absolute variance a component should have to be considered non-negligible (default is .Machine\$double.eps).

method	A character string specifying how the T-squared cutoffs are computed: "f" (default) or "beta". See Details.
conf.limit	A numeric vector of confidence levels, each strictly between 0 and 1, at which the T-squared cutoffs and ellipse semi-axes are computed (default is <code>c(0.95, 0.99)</code>). Any number of levels can be given. See Value for how the results are named.

Details

When `threshold` is used, the function selects the minimum number of k components that cumulatively explain at least the specified proportion of variance. This parameter allows for dynamic component selection based on explained variance, rather than using a fixed number of components. It must be greater than `rel.tol`. Typical values range from 0.8 to 0.95.

The `rel.tol` parameter sets a minimum variance threshold for individual components. Components with variance below this threshold are considered negligible and are removed from the analysis. Setting `rel.tol` too high may remove potentially important components, while setting it too low may retain noise or cause computational issues. Adjust based on your data characteristics and analysis goals.

Note that components are considered to have near-zero variance and are removed if their relative variance is below `rel.tol` or their absolute variance is below `abs.tol`. This dual-threshold approach helps ensure numerical stability while also accounting for the relative importance of components. The default value for `abs.tol` is set to `.Machine$double.eps`, providing a lower bound for detecting near-zero variance that may cause numerical instability.

The `method` parameter sets the distribution used for the cutoffs. With `method = "f"` (default), the cutoffs are $\frac{k(n-1)}{n-k} F_{\alpha}(k, n-k)$, as in previous versions of the package. With `method = "beta"`, the cutoffs follow the exact distribution of T-squared for the observations used to estimate the mean and covariance, $\frac{(n-1)^2}{n} B_{\alpha}(k/2, (n-k-1)/2)$ (Tracy, Young and Mason, 1992), e.g. the scores of the samples a PCA or PLS model was built on. The F-based limit is more conservative, especially for small n : it can even exceed the largest T-squared value any observation can reach, $(n-1)^2/n$. For n larger than about 100, the two limits are close.

When the selected components are correlated (e.g. new samples projected onto a model, or ICA scores), the ellipse is rotated so that it matches the T-squared statistic: an observation lies outside the ellipse exactly when its T-squared value exceeds the cutoff.

Value

A list containing the following elements:

- `Tsquare`: A data frame containing the T-squared statistic for each observation (the squared Mahalanobis distance), on the same scale as the cutoffs. When $k = 2$, it is computed on components `pcx` and `pcy`.
- `cutoff.<level>pct`: The T-squared cutoff value at each confidence level in `conf.limit`, from the highest to the lowest level. With the default `conf.limit = c(0.95, 0.99)`, these are `cutoff.99pct` and `cutoff.95pct`; with `conf.limit = c(0.975, 0.999)`, they are `cutoff.99.9pct` and `cutoff.97.5pct`.
- `nb.comp`: The number of components used in the calculation.

- **Ellipse:** A data frame containing the lengths of the semi-axes at each confidence level (a.<level>pct, b.<level>pct, e.g. a.99pct, b.99pct, a.95pct, b.95pct by default) and the rotation angle of the ellipse in radians (only when k = 2). a is the semi-axis closest to the pcx direction. For uncorrelated scores, such as the PCA or PLS scores of the samples the model was fitted on, angle is 0.

Author(s)

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References

Tracy, N. D., Young, J. C. and Mason, R. L. (1992). Multivariate control charts for individual observations. *Journal of Quality Technology*, 24(2), 88–95.

Examples

```
## Not run:
# Load required libraries
library(HotellingEllipse)
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# Perform PCA
set.seed(123)
pca_mod <- specData %>%
  select(where(is.numeric)) %>%
  FactoMineR::PCA(scale.unit = FALSE, graph = FALSE)

# Extract PCA scores
pca_scores <- pca_mod$ind$coord %>% as.data.frame()

# Example 1: Calculate Hotelling's T-squared and ellipse parameters using
# the 2nd and 4th components
T2_fixed <- ellipseParam(x = pca_scores, pcx = 2, pcy = 4)

# Example 2: Calculate using the first 4 components
T2_comp <- ellipseParam(x = pca_scores, k = 4)

# Example 3: Calculate using a cumulative variance threshold
T2_threshold <- ellipseParam(x = pca_scores, threshold = 0.95)

# Example 4: Cutoffs and semi-axes at the 97.5% and 99.9% confidence levels
T2_levels <- ellipseParam(x = pca_scores, conf.limit = c(0.975, 0.999))
T2_levels$cutoff.99.9pct
T2_levels$Ellipse$a.97.5pct

## End(Not run)
```

specData*LIBS spectra of 100 soil samples*

Description

Data set of the emission spectra of 100 soils measured in laboratory conditions. The samples were cleaned, dried, homogenized, sieved (10 Mesh size) and thereafter pelletized prior to LIBS measurements. LIBS spectra were preprocessed by performing baseline removal.

Usage

specData

Format

Data frame of 100 rows (soil samples) and 3152 columns (wavelengths).

Source

[doi:10.1039/C9JA00090A](https://doi.org/10.1039/C9JA00090A)

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